

# GRAPH THEORY WITH APPLICATIONS

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***To our parents***

# Preface

This book is intended as an introduction to graph theory. Our aim has been to present what we consider to be the basic material, together with a wide variety of applications, both to other branches of mathematics and to real-world problems. Included are simple new proofs of theorems of Brooks, Chvátal, Tutte and Vizing. The applications have been carefully selected, and are treated in some depth. We have chosen to omit all so-called 'applications' that employ just the language of graphs and no theory. The applications appearing at the end of each chapter actually make use of theory developed earlier in the same chapter. We have also stressed the importance of efficient methods of solving problems. Several good algorithms are included and their efficiencies are analysed. We do not, however, go into the computer implementation of these algorithms.

The exercises at the end of each section are of varying difficulty. The harder ones are starred (\*) and, for these, hints are provided in appendix I. In some exercises, new definitions are introduced. The reader is recommended to acquaint himself with these definitions. Other exercises, whose numbers are indicated by bold type, are used in subsequent sections; these should all be attempted.

Appendix II consists of a table in which basic properties of four graphs are listed. When new definitions are introduced, the reader may find it helpful to check his understanding by referring to this table. Appendix III includes a selection of interesting graphs with special properties. These may prove to be useful in testing new conjectures. In appendix IV, we collect together a number of unsolved problems, some known to be very difficult, and others more hopeful. Suggestions for further reading are given in appendix V.

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