

## AC network analysis

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Resources and methods for learning about these subjects (list a few here, in preparation for your research):

## Questions

### Question 1

Complex number arithmetic makes possible the analysis of AC circuits using (almost) the exact same Laws that were learned for DC circuit analysis. The only bad part about this is that doing complex-number arithmetic by hand can be very tedious. Some calculators, though, are able to add, subtract, multiply, divide, and invert complex quantities as easy as they do scalar quantities, making this method of AC circuit analysis relatively easy.

This question is really a series of practice problems in complex number arithmetic, the purpose being to give you lots of practice using the complex number facilities of your calculator (or to give you a *lot* of practice doing trigonometry calculations, if your calculator does not have the ability to manipulate complex numbers!).

#### Addition and subtraction:

$$(5 + j6) + (2 - j1) =$$

$$(10 - j8) + (4 - j3) =$$

$$(-3 + j0) + (9 - j12) =$$

$$(3 + j5) - (0 - j9) =$$

$$(25 - j84) - (4 - j3) =$$

$$(-1500 + j40) + (299 - j128) =$$

$$(25\angle 15^\circ) + (10\angle 74^\circ) =$$

$$(1000\angle 43^\circ) + (1200\angle -20^\circ) =$$

$$(522\angle 71^\circ) - (85\angle 30^\circ) =$$

#### Multiplication and division:

$$(25\angle 15^\circ) \times (12\angle 10^\circ) =$$

$$(1\angle 25^\circ) \times (500\angle -30^\circ) =$$

$$(522\angle 71^\circ) \times (33\angle 9^\circ) =$$

$$\frac{10\angle -80^\circ}{1\angle 0^\circ} =$$

$$\frac{25\angle 120^\circ}{3.5\angle -55^\circ} =$$

$$\frac{-66\angle 67^\circ}{8\angle -42^\circ} =$$

$$(3 + j5) \times (2 - j1) =$$

$$(10 - j8) \times (4 - j3) =$$

$$\frac{(3+j4)}{(12-j2)} =$$

#### Reciprocation:

$$\frac{1}{(15\angle 60^\circ)} =$$

$$\frac{1}{(750\angle -38^\circ)} =$$

$$\frac{1}{(10+j3)} =$$

$$\frac{1}{\frac{1}{15\angle 45^\circ} + \frac{1}{92\angle -25^\circ}} =$$

$$\frac{1}{\frac{1}{1200\angle 73^\circ} + \frac{1}{574\angle 21^\circ}} =$$

$$\frac{1}{\frac{1}{23k\angle -67^\circ} + \frac{1}{10k\angle -81^\circ}} =$$

$$\frac{1}{\frac{1}{110\angle -34^\circ} + \frac{1}{80\angle 19^\circ} + \frac{1}{70\angle 10^\circ}} =$$

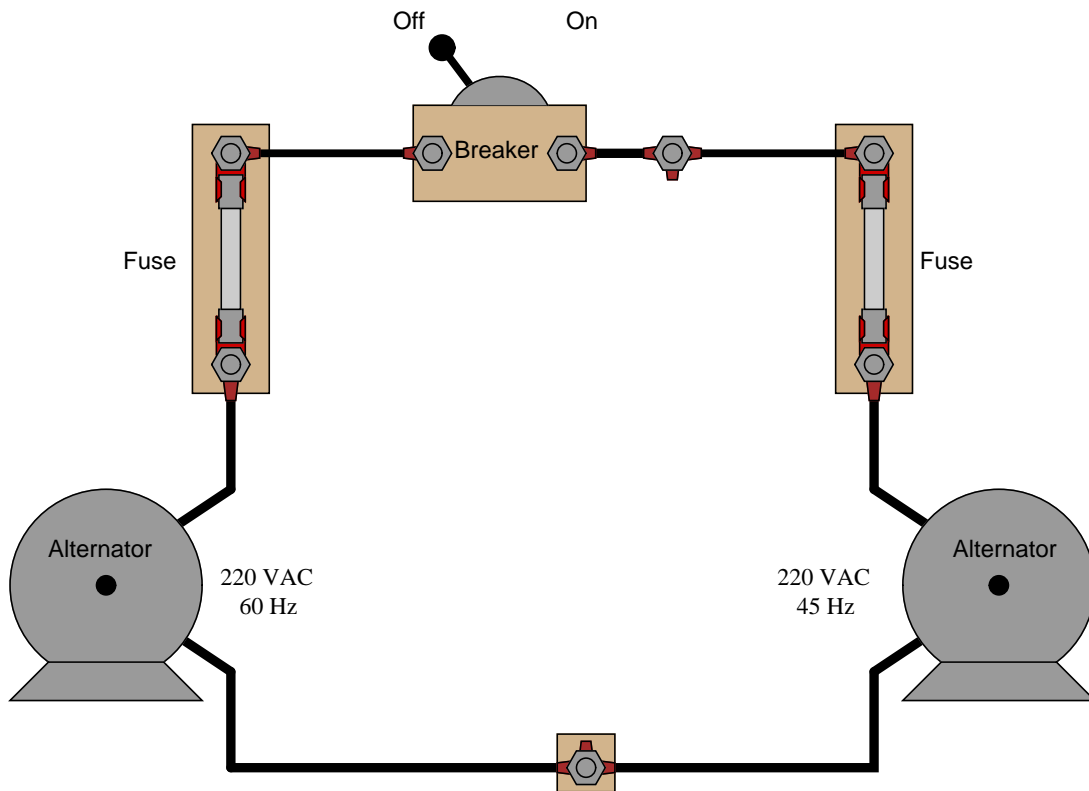
$$\frac{1}{\frac{1}{89k\angle -5^\circ} + \frac{1}{15k\angle 33^\circ} + \frac{1}{9.35k\angle 45^\circ}} =$$

$$\frac{1}{\frac{1}{512\angle 34^\circ} + \frac{1}{1k\angle -25^\circ} + \frac{1}{942\angle -20^\circ} + \frac{1}{2.2k\angle 44^\circ}} =$$

file 00860

Question 2

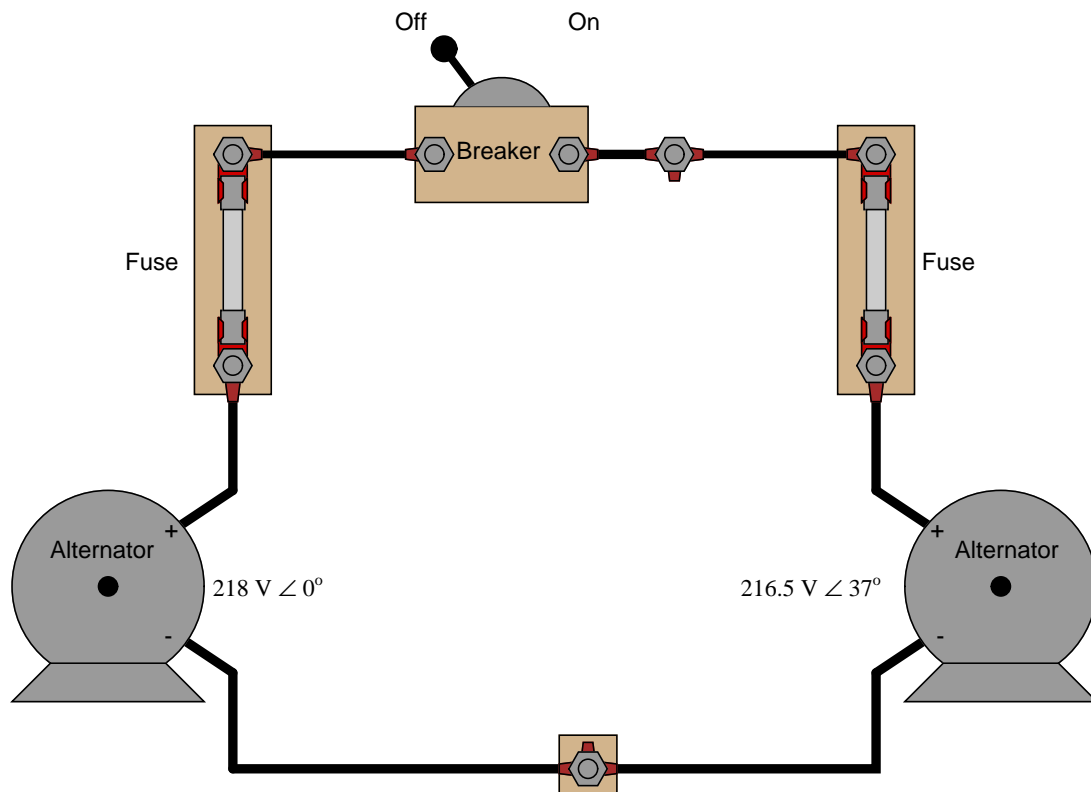
Is it safe to close the breaker between these two alternators if their output frequencies are different? Explain why or why not.



file 01057

Question 3

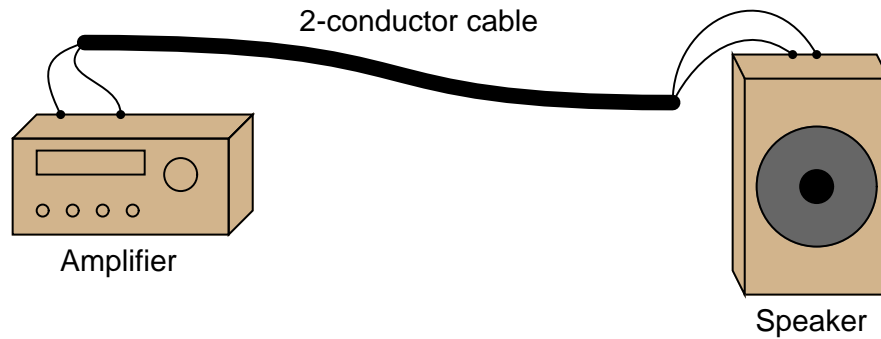
Given the output voltages of the two alternators, it is *not* safe to close the breaker. Explain why.



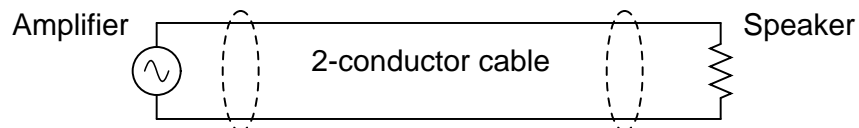
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Question 4

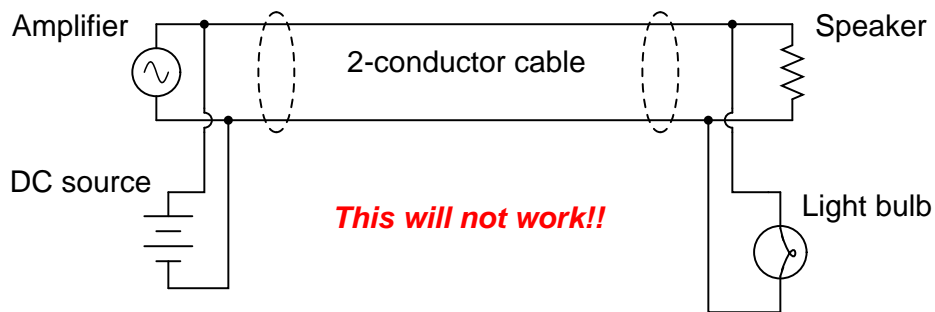
A remote speaker for an audio system is connected to the amplifier by means of a long, 2-conductor cable:



This system may be schematically modeled as an AC voltage source connected to a load resistor:

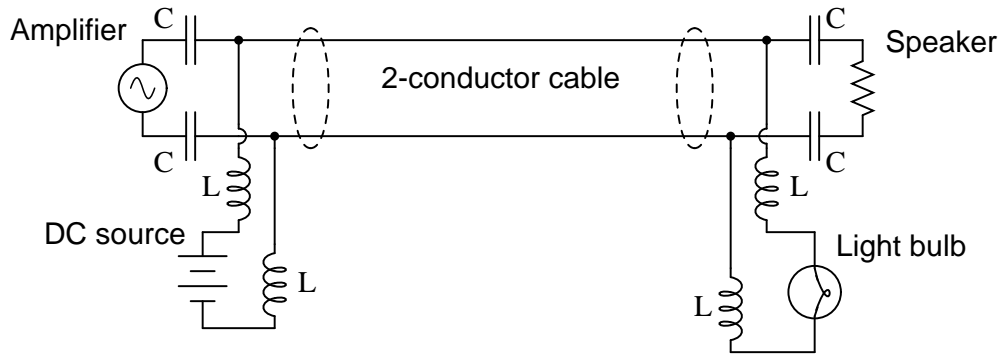


Suppose we decided to use the 2-conductor cable for more than just conveying an audio (AC) signal – we want to use it to carry DC power as well to energize a small lamp. However, if we were to simply connect the DC power source in parallel with the amplifier output at one end, and the lamp in parallel with the speaker at the other, bad things would happen:



If we were to connect the components together as shown above, the DC power source will likely damage the amplifier by being directly connected to it, the speaker will *definitely* be damaged by the application of significant DC voltage to its coil, and the light bulb will waste audio power by acting as a second (non-audible) load. Suffice to say, this is a bad idea.

Using inductors and capacitors as "filtering" components, though, we can make this system work:

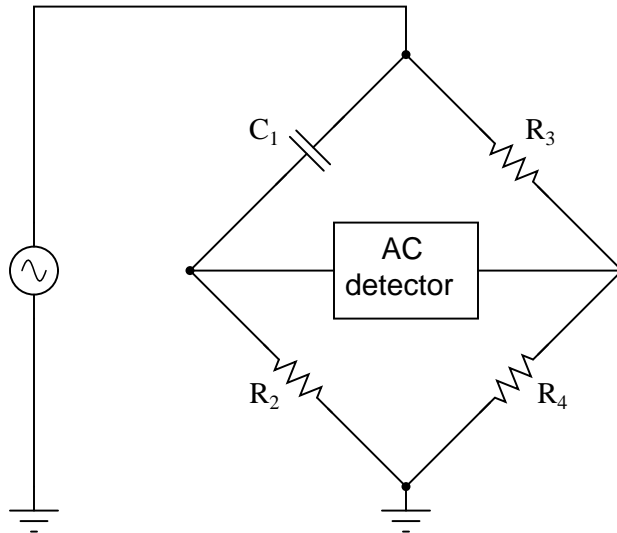


Apply the Superposition Theorem to this circuit to demonstrate that the audio and DC signals will not interfere with each other as they would if directly connected. Assume that the capacitors are of such large value that they present negligible impedance to the audio signal ( $Z_C \approx 0 \Omega$ ) and that the inductors are sufficiently large that they present infinite impedance to the audio signal ( $Z_L \approx \infty$ ).

[file 01856](#)

Question 5

Explain why this bridge circuit can *never* achieve balance:

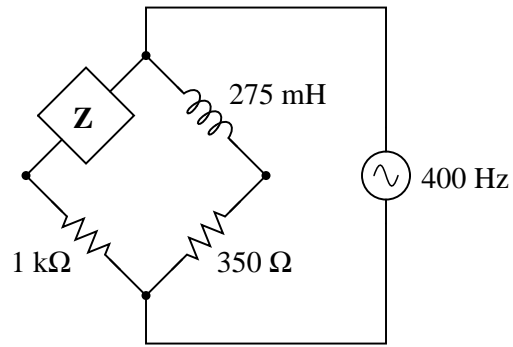


[file 01862](#)

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Question 6

Calculate the impedance value necessary to balance this AC bridge, expressing your answer in both polar and rectangular forms:

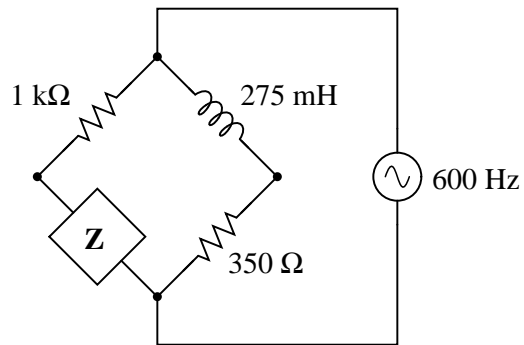


[file 01863](#)

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Question 7

Calculate the impedance value necessary to balance this AC bridge, expressing your answer in both polar and rectangular forms:



Also, describe what sort of device might be appropriate to serve as a "null detector" to indicate when bridge balance has been achieved, and where this device would be connected to in the bridge circuit.

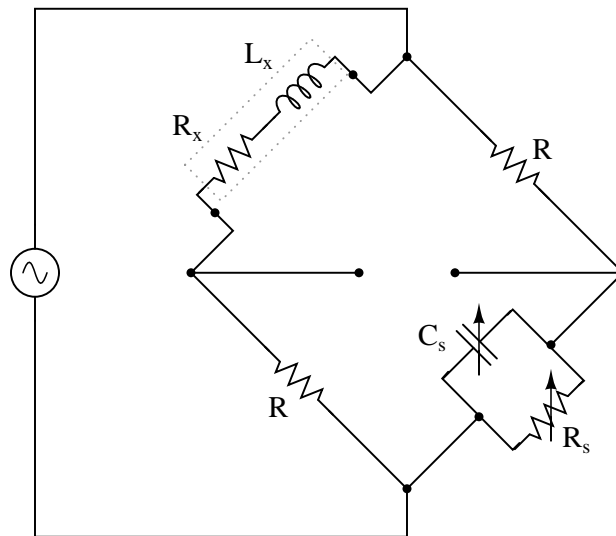
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Question 8

An AC bridge circuit commonly used to make precision measurements of inductors is the *Maxwell-Wien* bridge. It uses a combination of standard resistors and capacitors to "balance out" the inductor of unknown value in the opposite arm of the bridge:

*The Maxwell-Wien bridge*



Suppose this bridge circuit balances when  $C_s$  is adjusted to 120 nF and  $R_s$  is adjusted to 14.25 k $\Omega$ . If the source frequency is 400 Hz, and the two fixed-value resistors are 1 k $\Omega$  each, calculate the inductance ( $L_x$ ) and resistance ( $R_x$ ) of the inductor being tested.

[file 00862](#)

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Question 9

Electrical engineers often represent impedances in rectangular form for the sake of algebraic manipulation: to be able to construct and manipulate equations involving impedance, in terms of the components' fundamental values (resistors in ohms, capacitors in farads, and inductors in henrys).

For example, the impedance of a series-connected resistor ( $R$ ) and inductor ( $L$ ) would be represented as follows, with angular velocity ( $\omega$ ) being equal to  $2\pi f$ :

$$\mathbf{Z} = R + j\omega L$$

Using the same algebraic notation, represent each of the following complex quantities:

- Impedance of a single capacitor ( $C$ ) =
- Impedance of a series resistor-capacitor ( $R, C$ ) network =
- Admittance of a parallel inductor-resistor ( $L, R$ ) network =
- Admittance of a parallel resistor-capacitor ( $R, C$ ) network =

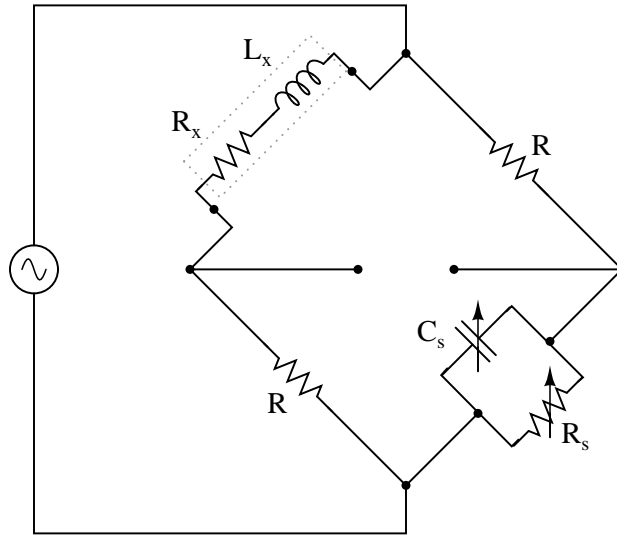
[file 02117](#)



Question 10

Mathematical analysis of the Maxwell-Wien bridge is as follows:

*The Maxwell-Wien bridge*



$$\mathbf{Z}_x = R_x + j\omega L_x \quad \text{Impedance of unknown inductance/resistance arm}$$

$$\mathbf{Z}_s = \frac{1}{\frac{1}{R_s} + \frac{1}{-j\omega C_s}} \quad \text{Impedance of standard capacitance/resistance arm}$$

$$\mathbf{Y}_s = \frac{1}{R_s} + j\omega C_s \quad \text{Admittance of standard capacitance/resistance arm}$$

$$\frac{\mathbf{Z}_x}{\mathbf{Z}_R} = \frac{\mathbf{Z}_R}{\mathbf{Z}_s} \quad \text{or} \quad \frac{\mathbf{Z}_x}{\mathbf{Z}_R} = \mathbf{Z}_R \mathbf{Y}_s \quad \text{Bridge balance equation}$$

$$\mathbf{Z}_x = R^2 \mathbf{Y}_s$$

$$R_x + j\omega L_x = R^2 \left( \frac{1}{R_s} + j\omega C_s \right)$$

$$R_x + j\omega L_x = \frac{R^2}{R_s} + j\omega R^2 C_s$$

Separating real and imaginary terms . . .

$$R_x = \frac{R^2}{R_s} \quad (\text{Real})$$

$$j\omega L_x = j\omega R^2 C_s \quad (\text{Imaginary})$$

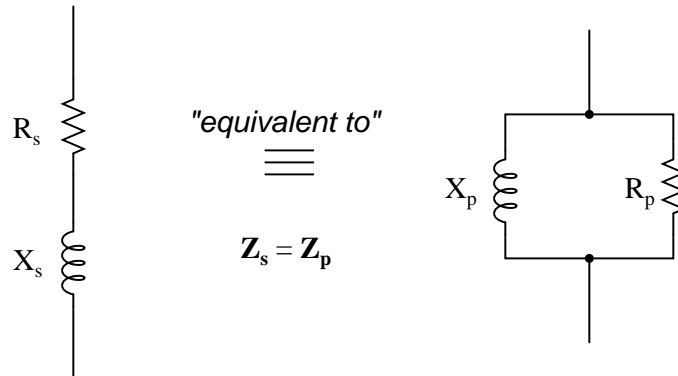
$$L_x = R^2 C_s$$

Note that neither of the two equations solving for unknown quantities ( $R_x = \frac{R_s^2}{R_p}$  and  $L_x = R_s^2 C_p$ ) contain the variable  $\omega$ . What does this indicate about the Maxwell-Wien bridge?

[file 01861](#)

Question 11

Suppose we have two equivalent LR networks, one series and one parallel, such that they have the exact same total impedance ( $Z_{\text{total}}$ ):



We may write an equation for the impedance of each network in rectangular form, like this:

$$Z_s = R_s + jX_s \quad (\text{series network})$$

$$Z_p = \frac{1}{\frac{1}{R_p} - j\frac{1}{X_p}} \quad (\text{parallel network})$$

Since we are told these two networks are equivalent to one another, with equal impedances, these two expressions in rectangular form must also be equal to each other:

$$R_s + jX_s = \frac{1}{\frac{1}{R_p} - j\frac{1}{X_p}}$$

Algebraically reduce this equation to its simplest form, showing how  $R_s$ ,  $R_p$ ,  $X_s$ , and  $X_p$  relate.

Challenge question: combine the result of that simplification with the equations solving for *scalar* impedance of series and parallel networks ( $Z_s^2 = R_s^2 + X_s^2$  for series and  $Z_p^2 = \frac{1}{\frac{1}{R_p^2} + \frac{1}{X_p^2}}$  for parallel) to prove the following transformative equations, highly useful for "translating" a series network into a parallel network and visa-versa:

$$Z^2 = R_p R_s$$

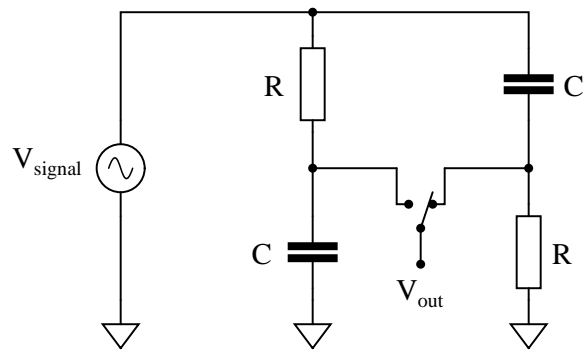
$$Z^2 = X_p X_s$$

[file 03291](#)

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Question 12

Determine the phase shift of the output voltage ( $V_{out}$ ) with reference to the source voltage ( $0^\circ$ ) for each of the two switch positions, assuming the source frequency is such that  $X_C = R$ :

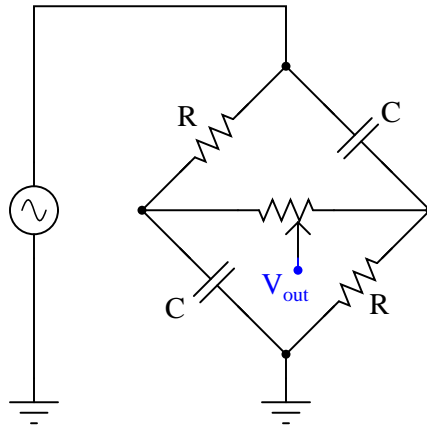


Note: you should be able to do all the necessary math mentally, without the aid of a calculating device!  
[file 03662](#)

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Question 13

This interesting bridge circuit is a variable phase-shifter. It works best when the excitation frequency is such that  $X_C = R$  in each arm of the bridge:



Supposing that  $X_C$  does equal  $R$  in each arm of the bridge, and that the potentiometer resistance is sufficiently high to limit current through it to a negligible level (in other words,  $R_{\text{pot}} \gg R$ ). Calculate the phase shift of  $V_{\text{out}}$  with respect to the excitation source voltage when:

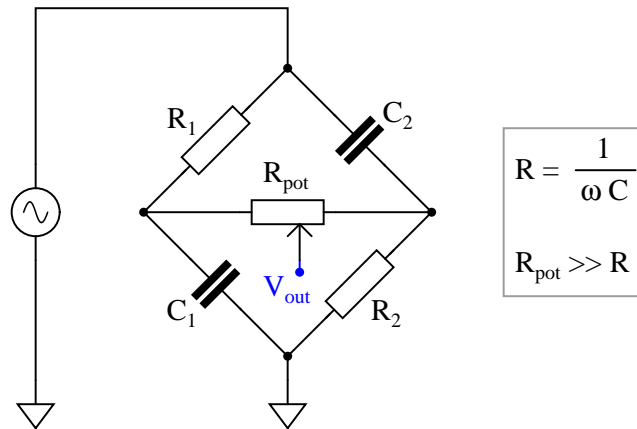
- The potentiometer wiper is fully left:
- The potentiometer wiper is fully right:
- The potentiometer wiper is perfectly centered:

[file 01599](#)

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Question 14

This phase-shifting bridge circuit is supposed to provide an output voltage with a variable phase shift from  $-45^\circ$  (lagging) to  $+45^\circ$  (leading), depending on the position of the potentiometer wiper:



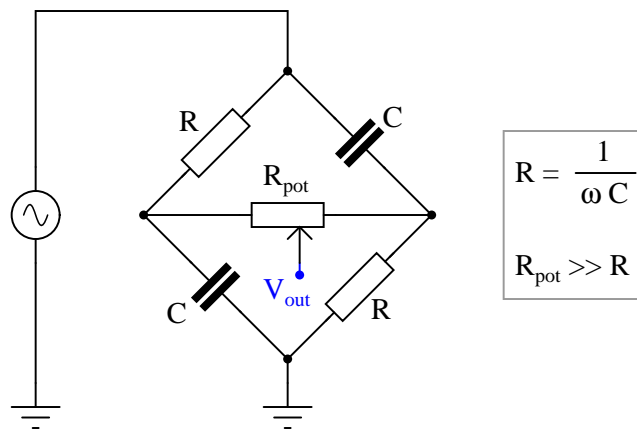
Suppose, though, that there is a solder "bridge" between the terminals of resistor  $R_1$  on the circuit board. What effect will this fault have on the output of the circuit? Be as complete as you can in your answer.

[file 03670](#)

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Question 15

This phase-shifting bridge circuit is supposed to provide an output voltage with a variable phase shift from  $-45^\circ$  (lagging) to  $+45^\circ$  (leading), depending on the position of the potentiometer wiper:



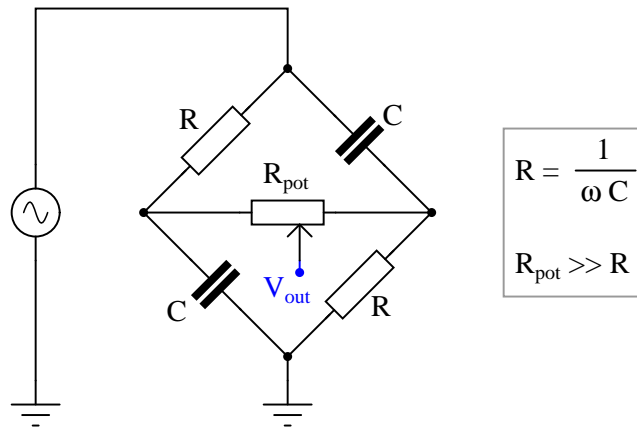
Suppose, though, that the output signal is stuck at  $+45^\circ$  leading the source voltage, no matter where the potentiometer is set. Identify a likely failure that could cause this to happen, and explain why this failure could account for the circuit's strange behavior.

[file 03464](#)

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Question 16

This phase-shifting bridge circuit is supposed to provide an output voltage with a variable phase shift from  $-45^\circ$  (lagging) to  $+45^\circ$  (leading), depending on the position of the potentiometer wiper:



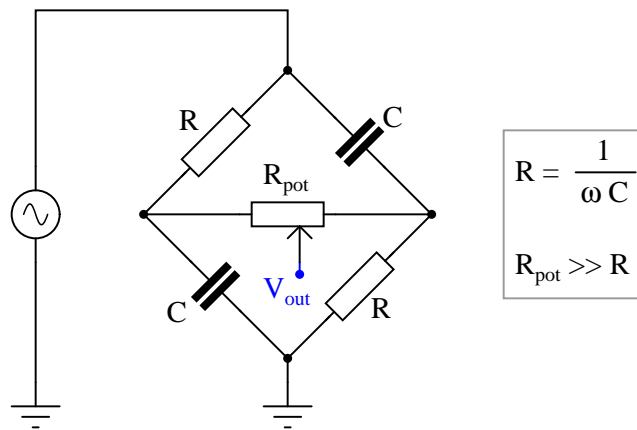
Suppose, though, that the output signal is stuck at  $-45^\circ$  lagging the source voltage, no matter where the potentiometer is set. Identify a likely failure that could cause this to happen, and explain why this failure could account for the circuit's strange behavior.

file 03465

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Question 17

This phase-shifting bridge circuit is supposed to provide an output voltage with a variable phase shift from  $-45^\circ$  (lagging) to  $+45^\circ$  (leading), depending on the position of the potentiometer wiper:



Suppose, though, that the output signal registers as it should with the potentiometer wiper fully to the right, but diminishes greatly in amplitude as the wiper is moved to the left, until there is practically zero output voltage at the full-left position. Identify a likely failure that could cause this to happen, and explain why this failure could account for the circuit's strange behavior.

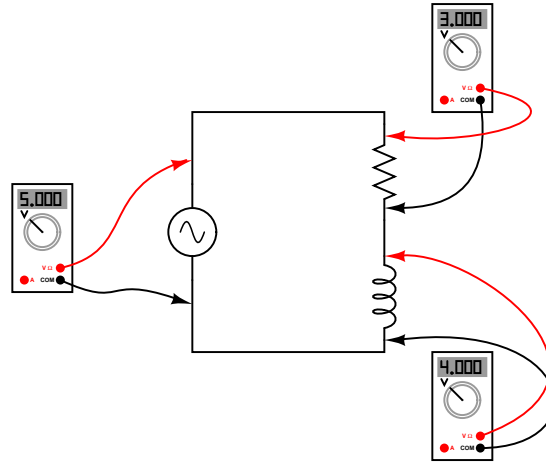
file 03466

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Question 18

Complex quantities may be expressed in either *rectangular* or *polar* form. Mathematically, it does not matter which form of expression you use in your calculations.

However, one of these forms relates better to real-world measurements than the other. Which of these mathematical forms (rectangular or polar) relates more naturally to measurements of voltage or current, taken with meters or other electrical instruments? For instance, which form of AC voltage expression, polar or rectangular, best correlates to the total voltage measurement in the following circuit?

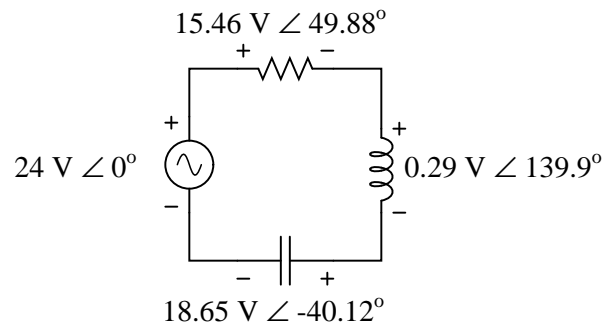


[file 01072](#)

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Question 19

Why are polarity marks (+ and -) shown at the terminals of the components in this AC network?



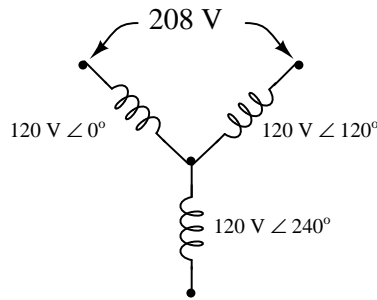
Are these polarity markings really necessary? Do they make any sense at all, given the fact that AC by its very nature has no fixed polarity (because polarity alternates over time)? Explain your answer.

[file 01054](#)

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Question 20

You should know that the line voltage of a three-phase, Y-connected, balanced system is always greater than the phase voltage by a factor of  $\sqrt{3}$ .



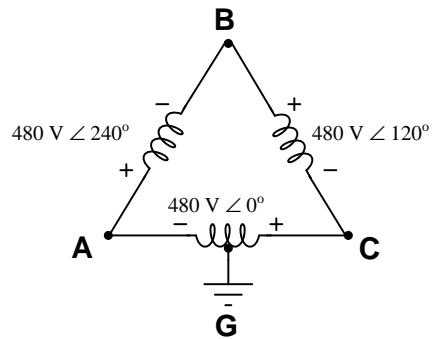
Apply Kirchhoff's Voltage Law (KVL) to the upper "loop" in this Y-connected alternator schematic to prove how  $120\text{ V} \angle 0^\circ$  and  $120\text{ V} \angle 120^\circ$  makes  $208\text{ V}$ . Show the "polarity" marks for each of the voltages as part of your answer.

[file 01055](#)

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Question 21

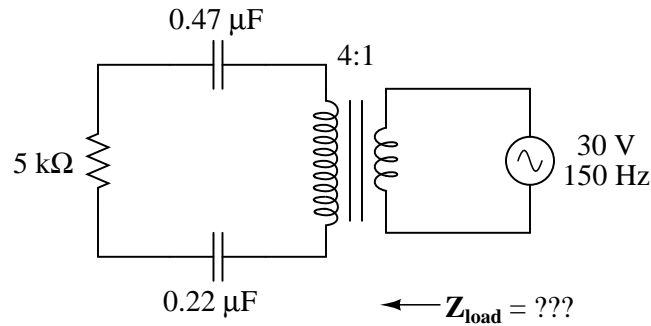
Draw a phasor diagram of the voltages in this Delta-connected polyphase system, and use trigonometry to calculate the voltage between points **G** and **B**.



[file 01059](#)

Question 22

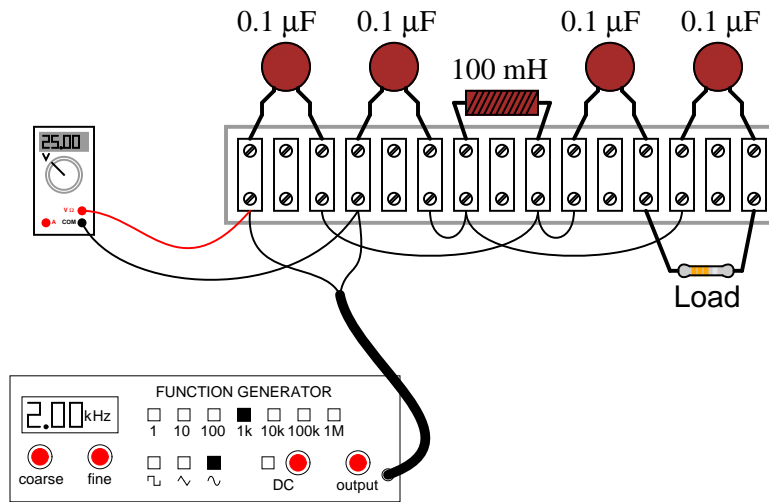
Calculate the load impedance "seen" by the source, through the ideal transformer with a winding turns ratio of 4:1.



[file 01060](#)

Question 23

Calculate all voltage drops in this circuit, expressing your answers in complex (polar) form:



The load resistor's color code is as follows:

**Brown, Black, Black, Brown, Violet**

Assume the resistor's error is 0%. That is, its resistance value is precisely equal to what the "digit" and "multiplier" color bands declare. The signal generator's output is 25 volts RMS, at a frequency of 2 kHz.

Challenge question: what practical function does this circuit perform?

[file 01071](#)



**Don't just sit there! Build something!!**

Learning to mathematically analyze circuits requires much study and practice. Typically, students practice by working through lots of sample problems and checking their answers against those provided by the textbook or the instructor. While this is good, there is a much better way.

You will learn much more by actually *building and analyzing real circuits*, letting your test equipment provide the "answers" instead of a book or another person. For successful circuit-building exercises, follow these steps:

1. Carefully measure and record all component values prior to circuit construction.
2. Draw the schematic diagram for the circuit to be analyzed.
3. Carefully build this circuit on a breadboard or other convenient medium.
4. Check the accuracy of the circuit's construction, following each wire to each connection point, and verifying these elements one-by-one on the diagram.
5. Mathematically analyze the circuit, solving for all voltage and current values.
6. Carefully measure all voltages and currents, to verify the accuracy of your analysis.
7. If there are any substantial errors (greater than a few percent), carefully check your circuit's construction against the diagram, then carefully re-calculate the values and re-measure.

For AC circuits where inductive and capacitive reactances (impedances) are a significant element in the calculations, I recommend high quality (high-Q) inductors and capacitors, and powering your circuit with low frequency voltage (power-line frequency works well) to minimize parasitic effects. If you are on a restricted budget, I have found that inexpensive electronic musical keyboards serve well as "function generators" for producing a wide range of audio-frequency AC signals. Be sure to choose a keyboard "voice" that closely mimics a sine wave (the "panflute" voice is typically good), if sinusoidal waveforms are an important assumption in your calculations.

As usual, avoid very high and very low resistor values, to avoid measurement errors caused by meter "loading". I recommend resistor values between 1 k $\Omega$  and 100 k $\Omega$ .

One way you can save time and reduce the possibility of error is to begin with a very simple circuit and incrementally add components to increase its complexity after each analysis, rather than building a whole new circuit for each practice problem. Another time-saving technique is to re-use the same components in a variety of different circuit configurations. This way, you won't have to measure any component's value more than once.

[file 00605](#)

## Answers

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Answer 1

**Addition and subtraction:**

$$(5 + j6) + (2 - j1) = \\ \mathbf{7 + j5}$$

$$(10 - j8) + (4 - j3) = \\ \mathbf{14 - j11}$$

$$(-3 + j0) + (9 - j12) = \\ \mathbf{6 - j12}$$

$$(3 + j5) - (0 - j9) = \\ \mathbf{3 + j14}$$

$$(25 - j84) - (4 - j3) = \\ \mathbf{21 - j81}$$

$$(-1500 + j40) + (299 - j128) = \\ \mathbf{-1201 - j88}$$

$$(25\angle 15^\circ) + (10\angle 74^\circ) = \\ \mathbf{31.35\angle 30.87^\circ}$$

$$(1000\angle 43^\circ) + (1200\angle -20^\circ) = \\ \mathbf{1878.7\angle 8.311^\circ}$$

$$(522\angle 71^\circ) - (85\angle 30^\circ) = \\ \mathbf{461.23\angle 77.94^\circ}$$

**Multiplication and division:**

$$(25\angle 15^\circ) \times (12\angle 10^\circ) = \\ \mathbf{300\angle 25^\circ}$$

$$(1\angle 25^\circ) \times (500\angle -30^\circ) = \\ \mathbf{500\angle -5^\circ}$$

$$(522\angle 71^\circ) \times (33\angle 9^\circ) = \\ \mathbf{17226\angle 80^\circ}$$

$$\frac{10\angle -80^\circ}{1\angle 0^\circ} = \\ \mathbf{10\angle -80^\circ}$$

$$\frac{25\angle 120^\circ}{3.5\angle -55^\circ} = \\ \mathbf{7.142\angle 175^\circ}$$

$$\frac{-66\angle 67^\circ}{8\angle -42^\circ} = \\ \mathbf{8.25\angle -71^\circ}$$

$$(3 + j5) \times (2 - j1) = \\ \mathbf{11 + j7}$$

$$(10 - j8) \times (4 - j3) = \\ \mathbf{16 - j62}$$

$$\frac{(3+j4)}{(12-j2)} = \\ \mathbf{0.1892 + j0.3649}$$

**Reciprocation:**

$$\frac{1}{(15\angle 60^\circ)} = \\ \mathbf{0.667\angle -60^\circ}$$

$$\frac{1}{(750\angle -38^\circ)} = \\ \mathbf{0.00133\angle 38^\circ}$$

$$\frac{1}{(10+j3)} = \\ \mathbf{0.0917 - j0.0275}$$

$$\frac{1}{\frac{1}{15\angle 45^\circ} + \frac{1}{92\angle -25^\circ}} = \\ \mathbf{14.06\angle 36.74^\circ}$$

$$\frac{1}{\frac{1}{1200\angle 73^\circ} + \frac{1}{574\angle 21^\circ}} = \\ \mathbf{425.7\angle 37.23^\circ}$$

$$\frac{1}{\frac{1}{23k\angle -67^\circ} + \frac{1}{10k\angle -81^\circ}} = \\ \mathbf{7.013k\angle -76.77^\circ}$$

$$\frac{1}{\frac{1}{110\angle -34^\circ} + \frac{1}{80\angle 19^\circ} + \frac{1}{70\angle 10^\circ}} = \\ \mathbf{29.89\angle 2.513^\circ}$$

$$\frac{1}{\frac{1}{89k\angle -5^\circ} + \frac{1}{15k\angle 33^\circ} + \frac{1}{9.35k\angle 45^\circ}} = \\ \mathbf{5.531k\angle 37.86^\circ}$$

$$\frac{1}{\frac{1}{512\angle 34^\circ} + \frac{1}{1k\angle -25^\circ} + \frac{1}{942\angle -20^\circ} + \frac{1}{2.2k\angle 44^\circ}} = \\ \mathbf{256.4\angle 9.181^\circ}$$

Answer 2

When the frequencies of two or more AC voltage sources are different, the phase shift(s) between them are constantly changing.

Follow-up question: what must be done to make the two alternators' frequencies equal to each other?

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Answer 3

The greatest problem with closing the breaker is the  $37^\circ$  phase shift between the two alternators' output voltages.

Follow-up question: what must be done to bring the two alternator voltages into phase with each other?

Challenge question: once the breaker is closed, can the two alternators ever fall out of phase with each other again?

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Answer 4

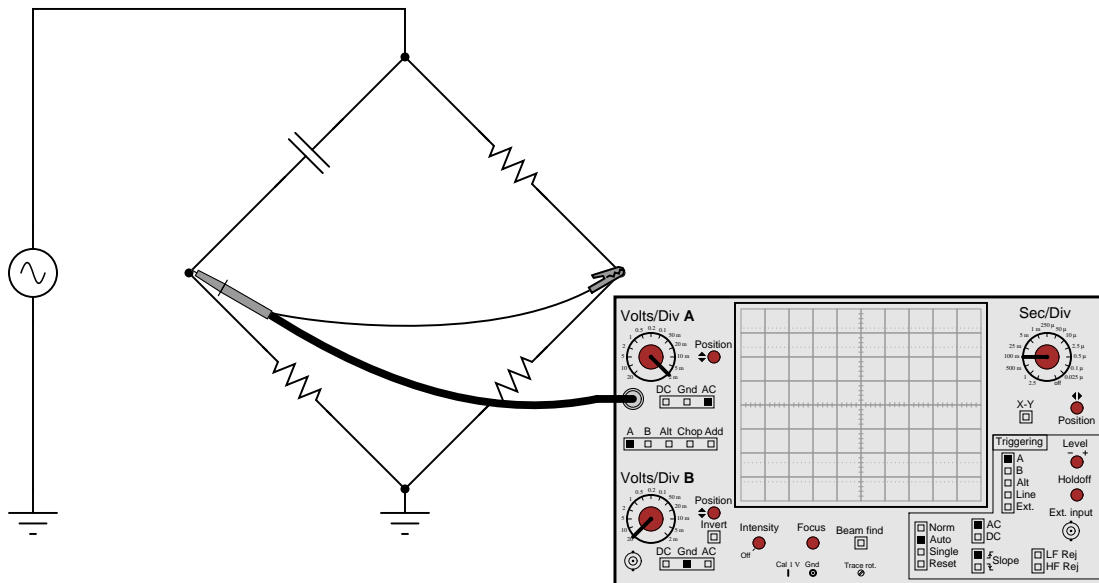
As each source is considered separately, the reactive components ensure each load receives the correct source voltage, with no interference.

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Answer 5

Even though impedance magnitudes may be balanced, the phase angles cannot.

Follow-up question: explain why it would *not* work to use an oscilloscope as the "detector" in this bridge circuit:



Hint: beware of *ground connections!*

Challenge question: explain how you *could* use this same oscilloscope as the detector in this bridge circuit without changing the ground location at the lower terminal of the AC source.

---

Answer 6

$$\mathbf{Z} = 1.975 \text{ k}\Omega \angle 90^\circ \text{ (polar form)}$$

$$\mathbf{Z} = 0 + j1.975 \text{ k}\Omega \text{ (rectangular form)}$$

Follow-up question: what type and size of component will provide this exact amount of impedance at 400 Hz?

---

Answer 7

$$\mathbf{Z} = 337.6 \Omega \angle -90^\circ \text{ (polar form)}$$

$$\mathbf{Z} = 0 - j337.6 \Omega \text{ (rectangular form)}$$

The simplest "null detector" for this type of AC bridge would be a sensitive pair of audio headphones, as 600 Hz is well within the audio range, and would be heard as a tone in the headphones.

Follow-up question: what type and size of component will provide this exact amount of impedance at 600 Hz?

---

Answer 8

$$L_x = 120 \text{ mH}$$

$$R_x = 70.175 \Omega$$

---

Answer 9

- Impedance of a single capacitor ( $C$ ) =  $-j\frac{1}{\omega C}$
- Impedance of a series resistor-capacitor ( $R, C$ ) network =  $R - j\frac{1}{\omega C}$
- Admittance of a parallel inductor-resistor ( $L, R$ ) network =  $\frac{1}{R} - j\frac{1}{\omega L}$
- Admittance of a parallel resistor-capacitor ( $R, C$ ) network =  $\frac{1}{R} + j\omega C$

---

Answer 10

This means the source frequency is irrelevant to bridge balance.

Due to the complexity of the algebra, I will show the complete solution here:

$$R_s + jX_s = \frac{1}{\frac{1}{R_p} - j\frac{1}{X_p}}$$

$$(R_s + jX_s) \left( \frac{1}{R_p} - j\frac{1}{X_p} \right) = 1$$

$$\frac{R_s}{R_p} - j\frac{R_s}{X_p} + j\frac{X_s}{R_p} - j^2\frac{X_s}{X_p} = 1$$

$$\frac{R_s}{R_p} - j\frac{R_s}{X_p} + j\frac{X_s}{R_p} + \frac{X_s}{X_p} = 1$$

$$\frac{R_s}{R_p} + \frac{X_s}{X_p} + j \left( \frac{X_s}{R_p} - \frac{R_s}{X_p} \right) = 1$$

*Separating real and imaginary terms . . .*

$$\frac{R_s}{R_p} + \frac{X_s}{X_p} = 1 \quad (\text{Real})$$

$$j \left( \frac{X_s}{R_p} - \frac{R_s}{X_p} \right) = j0 \quad (\text{Imaginary})$$

*Just working with the imaginary equation now . . .*

$$j \left( \frac{X_s}{R_p} - \frac{R_s}{X_p} \right) = j0$$

$$\frac{X_s}{R_p} - \frac{R_s}{X_p} = 0$$

$$\frac{X_s}{R_p} = \frac{R_s}{X_p}$$

$$X_p X_s = R_p R_s \quad (\text{Solution})$$

In answer to the challenge question, where we now introduce scalar relationships for series and parallel networks:

$$Z_s^2 = R_s^2 + X_s^2 \quad \text{Series impedance}$$

$$Z_p^2 = \frac{1}{\frac{1}{R_p^2} + \frac{1}{X_p^2}} \quad \text{Parallel impedance}$$

*Solving each scalar impedance equation for reactance X. . .*

$$X_s^2 = Z_s^2 - R_s^2 \quad X_p^2 = \frac{1}{\frac{1}{Z_p^2} - \frac{1}{R_p^2}}$$

Preparing the original solution for substitution . . .

$$X_p X_s = R_p R_s$$

$$(X_p X_s)^2 = (R_p R_s)^2$$

$$X_p^2 X_s^2 = R_p^2 R_s^2$$

Substituting these definitions for reactance into this prepared equation . . .

$$\left( \frac{1}{\frac{1}{Z_p^2} - \frac{1}{R_p^2}} \right) (Z_s^2 - R_s^2) = R_p^2 R_s^2$$

$$Z_s^2 - R_s^2 = (R_p^2 R_s^2) \left( \frac{1}{Z_p^2} - \frac{1}{R_p^2} \right)$$

$$Z_s^2 - R_s^2 = \frac{R_p^2 R_s^2}{Z_p^2} - \frac{R_p^2 R_s^2}{R_p^2}$$

$$Z_s^2 - R_s^2 = \frac{R_p^2 R_s^2}{Z_p^2} - R_s^2$$

$$Z_s^2 = \frac{R_p^2 R_s^2}{Z_p^2}$$

$$Z_p^2 Z_s^2 = R_p^2 R_s^2$$

$$(Z_p Z_s)^2 = (R_p R_s)^2$$

$$\sqrt{(Z_p Z_s)^2} = \sqrt{(R_p R_s)^2}$$

$$Z_p Z_s = R_p R_s$$

Since the two networks are known to be equivalent,  $Z_p = Z_s$ , which I will now simply label as  $Z$ . . .

$$ZZ = R_p R_s$$

$$Z^2 = R_p R_s \quad \text{(Solution)}$$

And since we know that  $R_p R_s = X_p X_s$  as well . . .

$$Z^2 = X_p X_s \quad \text{(Solution)}$$

Follow-up question: the original equivalent networks were comprised of a resistor ( $R$ ) and an inductor ( $L$ ). Show that these solutions ( $Z^2 = R_p R_s$  and  $Z^2 = X_p X_s$ ) hold true for resistor-*capacitor* series and parallel equivalent networks as well.

---

Answer 12

Switch left:  $\Theta = -45^\circ$  ( $V_{out}$  lagging behind the source voltage)

Switch right:  $\Theta = +45^\circ$  ( $V_{out}$  leading ahead of the source voltage)

Follow-up question: identify the effects of various component failures in this circuit.

---

Answer 13

- The potentiometer wiper is fully left:  $\Theta = -45^\circ$
- The potentiometer wiper is fully right:  $\Theta = 45^\circ$
- The potentiometer wiper is perfectly centered:  $\Theta = 0^\circ$

Follow-up question: this circuit works best with the excitation frequency is such that  $X_C = R$ . Write a formula that solves for the necessary frequency ( $f$ ) to achieve this condition given a certain value of  $R$ .

---

Answer 14

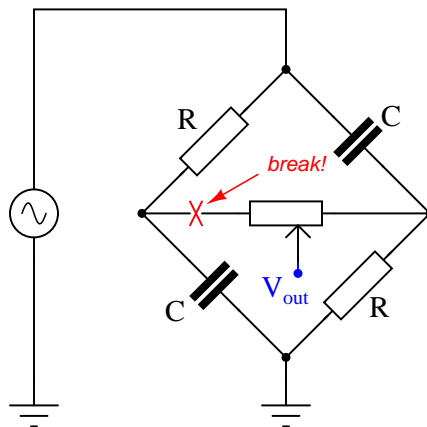
With such a "shorted" failure on  $R_1$ , there will be full source voltage at the output with the potentiometer wiper at the full-left position (no attenuation, no phase shift). The output voltage at the full-right wiper position will be mostly unaffected.

Follow-up question: identify another possible component failure that would exhibit the same symptoms.

---

Answer 15

A broken connection between the left-hand terminal of the potentiometer and the bridge could cause this to happen:

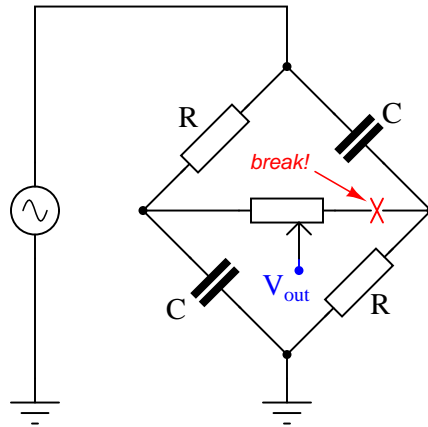


I'll let you figure out why!

---

Answer 16

A broken connection between the right-hand terminal of the potentiometer and the bridge could cause this to happen:

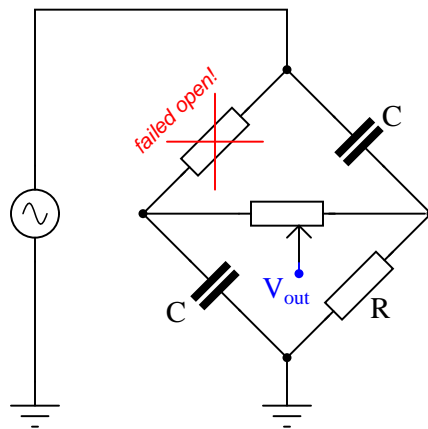


I'll let you figure out why!

---

Answer 17

An open failure of the fixed resistor in the upper-left arm of the bridge could cause this to happen:



Follow-up question: identify another possible component failure that would exhibit the same symptoms.

---

Answer 18

Polar form relates much better to the voltmeter's display of 5 volts.

Follow-up question: how would you represent the total voltage in this circuit in rectangular form, given the other two voltmeter readings?

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Answer 19

The polarity markings provide a frame of reference for the phase angles of the voltage drops.

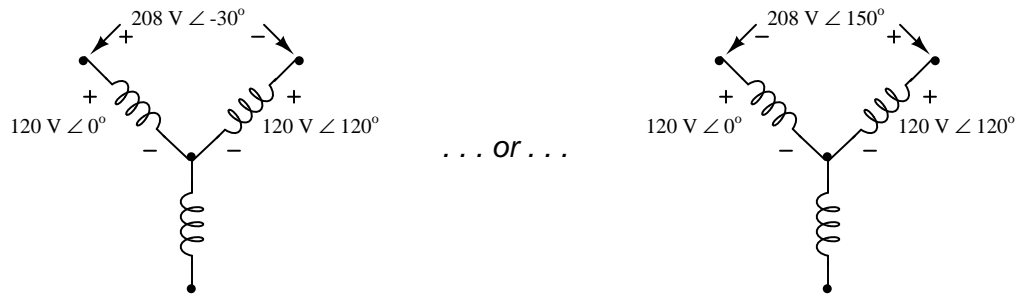


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Answer 20

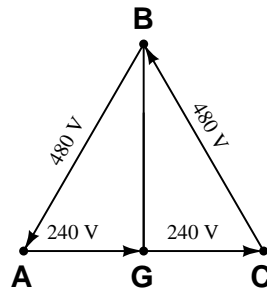
$$(120 \text{ V} \angle 0^\circ) - (120 \text{ V} \angle 120^\circ) = 208 \text{ V} \angle -30^\circ$$

$$(120 \text{ V} \angle 120^\circ) - (120 \text{ V} \angle 0^\circ) = 208 \text{ V} \angle 150^\circ$$



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Answer 21

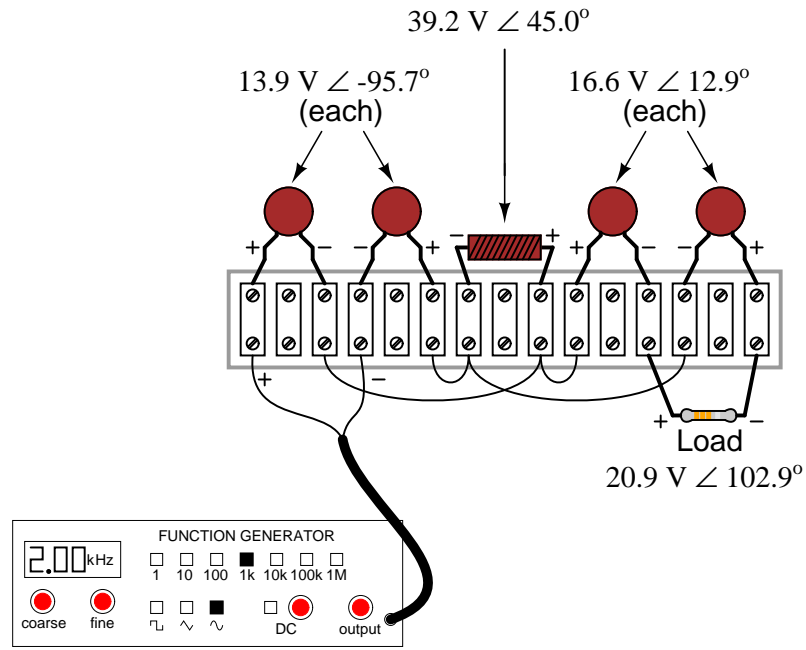


$$V_{BG} = 415.7 \text{ V}$$

---

Answer 22

$$\mathbf{Z}_{\text{load}} = 541.7 \Omega \angle -54.77^\circ$$



This circuit is a high-pass filter.

Let the electrons themselves give you the answers to your own "practice problems"!

## Notes

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### Notes 1

I suggest you let your students discover how to use the complex number facilities of their scientific calculators on their own. My experience has been that students both young and old take to this challenge readily, because they realize learning how to use their calculators will save them a tremendous amount of hand calculations!

---

### Notes 2

Ask your students to calculate how fast the voltages from these two alternators "roll" in and out of phase with each other.

---

### Notes 3

Discuss the consequences of closing the breaker when there is such a large phase shift between the two alternator output voltages. What will likely happen in the circuit if the breaker is closed under these conditions?

Ask your students whether or not the discrepancy in output voltage (218 VAC versus 216.5 VAC) is of any consequence in closing the breaker. Why is phase shift the only factor mentioned in the answer as a reason not to close the breaker?

This question serves to illustrate alternator theory as well as AC network analysis principles. The "phase-locking" phenomenon of two paralleled alternators is very important for students to understand if they are to do any work related to AC power generation systems.

---

### Notes 4

Such "power-plus-data" strategies are made possible by the Superposition Theorem and the linearity of resistors, capacitors, and inductors. If time permits, this would be a good opportunity to discuss "power-line carrier" systems, where high-frequency data is transmitted over power line conductors. The venerable X10 network system is an example for residential power wiring, while power distribution utilities have been using this "PLC" technology (the acronym not to be confused with *Programmable Logic Controllers*) for decades over high-voltage transmission lines.

---

### Notes 5

Explain to your students that AC bridges, while fundamentally the same as DC bridges, do have their differences. Their knowledge of AC circuit calculations (with complex numbers) should be enough for them to see why this is so.

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### Notes 6

So long as complex quantities are used, AC bridge circuits "balance" just the same way that DC bridge circuits balance. Consequently, this is really nothing new for your students if they've already studied DC Wheatstone bridge circuits.

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### Notes 7

So long as complex quantities are used, AC bridge circuits "balance" just the same way that DC bridge circuits balance. Consequently, this is really nothing new for your students if they've already studied DC Wheatstone bridge circuits.

---

Notes 8

There is actually a way to solve for the values of  $L_x$  and  $R_x$  in a Maxwell-Wien bridge circuit without using complex numbers at all. If one or more of your students find out how to do so through their research, don't consider it "cheating." Rather, applaud their research, because they found a quicker path to a solution.

This, of course, doesn't mean you don't ask them to work through the problem using complex numbers! The benefit of students researching other means of solution simply provides more alternative solution strategies to the same problem, which is a very good thing.

---

Notes 9

One possible point of confusion here is the sign of  $j$  after inversion. If it is not evident from the answers,  $\frac{1}{j}$  is equal to  $-j$ , so that the impedance of an inductor ( $j\omega L$ ) becomes  $-j\frac{1}{\omega L}$  when converted into an admittance.

---

Notes 10

A rule that students will need to be aware of in order to follow all the algebra shown here is  $\frac{1}{j} = -j$ . A brief proof is given here:

$$j^2 = -1$$

$$\frac{1}{j^2} = \frac{1}{-1}$$

$$\frac{j}{j^2} = \frac{j}{-1}$$

$$\frac{j}{j^2} = -j$$

$$\frac{1}{j} = -j$$

Otherwise, the admittance equation ( $\mathbf{Y}_s = \frac{1}{R_s} + j\omega C_s$ ) will not make sense.

One type of AC bridge is called a *noise bridge*, using a "noise" (broadly mixed frequencies) source as the excitation voltage. Discuss with your students how the Maxwell-Wien bridge would be a suitable topology for use with a noise source, whereas other bridge topologies might not.

---

Notes 11

Yes, it is out of character for me to show two pages of solution in the "answer" section of one of my questions! I usually do not provide this much information in my answers. However, in this case I believe there is still much to be learned from examining a proof like this shown step-by-step.

You may wish to ask your students to explain the rationale behind each step, especially in the first part where we deal with real and imaginary terms. One point that may be especially confusing is where I separate the real and imaginary terms, setting the imaginary quantity equal to  $j0$ . Some students may not see where the  $j0$  comes from, since the preceding (complex) expression was simply equal to 1. Remind them that "1" is a real quantity, possessing an (implied) imaginary component of 0, and that it could very well have been written as  $1 + j0$ .

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Notes 12

This is a very interesting circuit to build and test. You may build one using 1  $\mu\text{F}$  capacitors and 2.7 k $\Omega$  resistors that will successfully operate on 60 Hz power-line excitation.

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#### Notes 13

This is a very interesting circuit to built and test. You may build one using 1  $\mu\text{F}$  capacitors, 2.7 k $\Omega$  resistors, and a 100 k $\Omega$  potentiometer that will successfully operate on 60 Hz power-line excitation.

An interesting thing to note about using line power is that any distortions in the excitation sine-wave will become obvious when the potentiometer wiper is turned toward the differentiating position (where  $\Theta$  is positive). If listened to with an audio detector, you may even hear the change in timbre while moving the wiper from one extreme to the other. If excited by a "clean" sine-wave, however, no change in timbre should be heard because there are no harmonics present.

---

#### Notes 14

It is essential, of course, that students understand the operational principle of this circuit before they may speculate at the effects of various component faults. You may find it necessary to discuss this circuit in detail with your students before they are ready to troubleshoot it.

In case anyone asks, the symbolism  $R_{pot} \gg R$  means "potentiometer resistance is *much* greater than the fixed resistance value."

---

#### Notes 15

It is essential, of course, that students understand the operational principle of this circuit before they may even attempt to diagnose possible faults. You may find it necessary to discuss this circuit in detail with your students before they are ready to troubleshoot it.

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---

#### Notes 16

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---

#### Notes 17

It is essential, of course, that students understand the operational principle of this circuit before they may even attempt to diagnose possible faults. You may find it necessary to discuss this circuit in detail with your students before they are ready to troubleshoot it.

In case anyone asks, the symbolism  $R_{pot} \gg R$  means "potentiometer resistance is *much* greater than the fixed resistance value."

---

#### Notes 18

While rectangular notation is mathematically useful, it does not apply directly to measurements taken with real instruments. Some students might suggest that the 3.000 volt reading and the 4.000 volt reading on the other two voltmeters represent the rectangular components (real and imaginary, respectively) of voltage, but this is a special case. In cases where resistance and reactance are mixed (e.g. a practical inductor with winding resistance), the voltage magnitude will be neither the real nor the imaginary component, but rather the polar magnitude.

---

#### Notes 19

Ask your students why polarity markings need to be provided in DC electrical networks, as an essential part of the voltage figures. Why is an answer for a voltage drop incomplete if not accompanied by polarity markings in a DC circuit? Discuss this with your students, then ask them to extrapolate this principle to AC circuits. When we are accounting for the *phase shift* of a voltage drop in our answer, does the "polarity" of the voltage drop matter?

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Notes 20

This question is highly effective in demonstrating why polarity markings are important in AC circuit analysis. Without the polarity marks as "frames of reference" for the phase angles, it is impossible to determine the resultant line voltage from the two 120 VAC phase voltages.

---

Notes 21

Of course, complex numbers may also be used to calculate  $V_{AB}$ , but the phasor approach has the benefit of graphical intuitiveness.

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Notes 22

Ask your students to describe the effect of the transformer's step-up ratio on impedance magnitude and on impedance phase angle. They may do this by comparing the impedance of the C-R-C circuit on the transformer's secondary side versus the impedance "seen" at the voltage source terminals. Ask them to explain why one of these parameters is affected but the other is not.

Then, pose the scenario of a real transformer, complete with leakage inductance. Ask your students to explain what effect would leakage inductance have on the load impedance of this circuit.

---

Notes 23

The real "answer" to this question is the circuit analysis technique employed. Ask your students, perhaps working teams, to explain how to arrive at the different voltage drop figures. Note any similarities in analysis technique to that of DC circuit analysis.

It has been my experience that students require much practice with circuit analysis to become proficient. To this end, instructors usually provide their students with lots of practice problems to work through, and provide answers for students to check their work against. While this approach makes students proficient in circuit theory, it fails to fully educate them.

Students don't just need mathematical practice. They also need real, hands-on practice building circuits and using test equipment. So, I suggest the following alternative approach: students should *build* their own "practice problems" with real components, and try to mathematically predict the various voltage and current values. This way, the mathematical theory "comes alive," and students gain practical proficiency they wouldn't gain merely by solving equations.

Another reason for following this method of practice is to teach students *scientific method*: the process of testing a hypothesis (in this case, mathematical predictions) by performing a real experiment. Students will also develop real troubleshooting skills as they occasionally make circuit construction errors.

Spend a few moments of time with your class to review some of the "rules" for building circuits before they begin. Discuss these issues with your students in the same Socratic manner you would normally discuss the worksheet questions, rather than simply telling them what they should and should not do. I never cease to be amazed at how poorly students grasp instructions when presented in a typical lecture (instructor monologue) format!

An excellent way to introduce students to the mathematical analysis of real circuits is to have them first determine component values (L and C) from measurements of AC voltage and current. The simplest circuit, of course, is a single component connected to a power source! Not only will this teach students how to set up AC circuits properly and safely, but it will also teach them how to measure capacitance and inductance without specialized test equipment.

A note on reactive components: use high-quality capacitors and inductors, and try to use low frequencies for the power supply. Small step-down power transformers work well for inductors (at least two inductors in one package!), so long as the voltage applied to any transformer winding is less than that transformer's rated voltage for that winding (in order to avoid saturation of the core).

A note to those instructors who may complain about the "wasted" time required to have students build real circuits instead of just mathematically analyzing theoretical circuits:

*What is the purpose of students taking your course?*

If your students will be working with real circuits, then they should learn on real circuits whenever possible. If your goal is to educate theoretical physicists, then stick with abstract analysis, by all means! But most of us plan for our students to do something in the real world with the education we give them. The "wasted" time spent building real circuits will pay huge dividends when it comes time for them to apply their knowledge to practical problems.

Furthermore, having students build their own practice problems teaches them how to perform *primary research*, thus empowering them to continue their electrical/electronics education autonomously.

In most sciences, realistic experiments are much more difficult and expensive to set up than electrical circuits. Nuclear physics, biology, geology, and chemistry professors would just love to be able to have their students apply advanced mathematics to real experiments posing no safety hazard and costing less than a textbook. They can't, but you can. Exploit the convenience inherent to your science, and *get those students of yours practicing their math on lots of real circuits!*