CSCI 1900
Discrete Structures

Combinations
Reading: Kolman, Section 3.2

Order Doesn't Matter

In the previous section, we looked at two cases where order matters:

- Multiplication Principle – duplicates allowed
- Permutations – duplicates not allowed

Order Doesn't Matter
Duplicates Not Allowed

- What if order doesn't matter, for example, a hand of cards in poker?
- Example: the elements 6, 5, and 2 make six possible sequences: 652, 625, 256, 265, 526, and 562
- If order doesn't matter, these six sequences would be considered the same.

Removing Order from Order

Notice the example given on the previous slide of the possible sequences involving the elements 6, 5, and 2. The number of arrangements of 6, 5, and 2 equals the number of ways three elements can be ordered, i.e., ${}_3P_3$.

$${}_3P_3 = 3!/(3-3)! = 6/1 = 6$$

Removing Order from Order (continued)

- Assume that we came up with the number of permutations of three elements from the ten decimal digits

$${}_{10}P_3 = 10!/(10-3)! = 10!/7! = 720$$

- Each subset of three integers from the ten decimal digits would produce 6 sequences.
- Therefore, to remove order from the 720 sequences, simply divide by 6 to get 120.

Combinations of 3 Digits

012	027	048	123	139	169	247	289	369	479
013	028	049	124	145	178	248	345	378	489
014	029	056	125	146	179	249	346	379	567
015	034	057	126	147	189	256	347	389	568
016	035	058	127	148	234	257	348	456	569
017	036	059	128	149	235	258	349	457	578
018	037	067	129	156	236	259	356	458	579
019	038	068	134	157	237	267	357	459	589
023	039	069	135	158	238	268	358	467	678
024	045	078	136	159	239	269	359	468	679
025	046	079	137	167	245	278	367	469	689
026	047	089	138	168	246	279	368	478	789

Combinations

- Notation: ${}_nC_r$ is called number of combinations of n objects taken r at a time.

$${}_nC_r = n!/[r! \cdot (n - r)!]$$

- Example: How many 5 card hands can be dealt from a deck of 52?

$${}_{52}C_5 = 52!/(5! \cdot (52\text{-}5)!)$$

- Example: Pick 3 horses from 10 to place in any order
- Why are these examples different?
  - How many ways can a pair of dice come up?
  - How many dominoes are there in a pack?

Order Doesn't Matter Duplicates Allowed

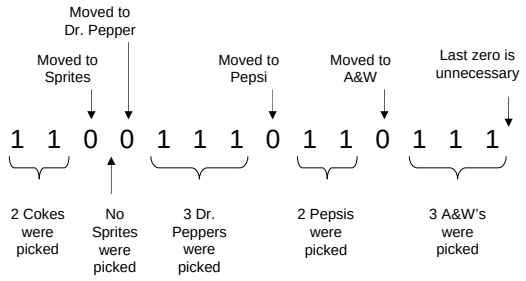
Assume you are walking with your grocery cart past the 2 liter sodas in Walmart. You need to pick up 10 bottles out of:

- Coke
- Sprite
- Dr. Pepper
- Pepsi
- A&W Root Beer

Buying Sodas

- You can define how you selected the sodas with a binary string of ones and zeros.
- A one indicates you have selected a soda from that category. A zero says that you have moved onto the next category.

Buying Sodas (continued)



Buying Sodas (continued)

- This means that a binary pattern of 10 + (5 – 1) = 14 ones and zeros can be used to represent a selection of 10 items from 5 possibilities without worrying about order and allowing duplicates.
- This is the same as having 14 elements from which we will select 10 to be set as one, i.e.,

$${}_{14}C_{10} = 14!/(10! \cdot (14 - 10)!) = 1001$$

Order Doesn't Matter Duplicates Allowed

The general formula for order doesn't matter and duplicates allowed for a selection of *r* items from a set of *n* items is:

$${}_{(n + r - 1)}C_r$$