CSCI 1900
Discrete Structures

Conditional Statements
Reading: Kolman, Section 2.2

Conditional Statement/Implication

- "if *p* then *q*"
- Denoted $p \Rightarrow q$
  - p is called the ***antecedent*** or ***hypothesis***
  - q is called the ***consequent*** or ***conclusion***
- Example:
  - *p*: I am hungry
    q: I will eat
  - *p*: It is snowing
    q: 3+5 = 8

Conditional Statement/Implication (continued)

- In English, we would assume a cause-and-effect relationship, i.e., the fact that p is true would force q to be true.
- If "it is snowing," then "3+5=8" is meaningless in this regard since *p* has no effect at all on *q*
- At this point it may be easiest to view the operator "⇒" as a logic operationsimilar to AND or OR (conjunction or disjunction).

Truth Table Representing Implication

- If viewed as a logic operation, $p \Rightarrow q$ can only be evaluated as false if ***p is true and q is false***
- This does not say that *p* causes *q*
- Truth table

p	q	$p \Rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Examples where $p \Rightarrow q$ is viewed as a logic operation

- If *p* is false, then any *q* supports $p \Rightarrow q$ is true.
  - False ⇒ True = True
  - False ⇒ False = True
- If "2+2=5" then "I am the king of England" is true

Converse and contrapositive

- The converse of $p \Rightarrow q$ is the implication that $q \Rightarrow p$
- The contrapositive of $p \Rightarrow q$ is the implication that $\sim q \Rightarrow \sim p$

Converse and Contrapositive Example

Example: What is the converse and contrapositive of *p*: "it is raining" and *q*: I get wet?

- Implication: If it is raining, then I get wet.
- Converse: If I get wet, then it is raining.
- Contrapositive: If I do not get wet, then it is not raining.

Equivalence or biconditional

- If *p* and *q* are statements, the compound statement *p* ***if and only if*** *q* is called an ***equivalence*** or ***biconditional***
- Denoted $p \Leftrightarrow q$

Equivalence Truth table

- The only time that the expression can evaluate as true is if both statements, p and q, are true or both are false

p	*Q*	$p \Leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Proof of the Contrapositive

Compute the truth table of the statement $(p \Rightarrow q) \Leftrightarrow (\sim q \Rightarrow \sim p)$

p	q	$p \Rightarrow q$	$\sim q$	$\sim p$	$\sim q \Rightarrow \sim p$	$(p \Rightarrow q) \Leftrightarrow (\sim q \Rightarrow \sim p)$
T	T	T	F	F	T	T
T	F	F	T	F	F	T
F	T	T	F	T	T	T
F	F	T	T	T	T	T

Tautology and Contradiction

- A statement that is true for all of its propositional variables is called a ***tautology***. (The previous truth table was a tautology.)
- A statement that is false for all of its propositional variables is called a ***contradiction*** or an ***absurdity***

Contingency

- A statement that can be either true or false depending on its propositional variables is called a ***contingency***
- Examples
  - $(p \Rightarrow q) \Leftrightarrow (\sim q \Rightarrow \sim p)$ is a tautology
  - $p \wedge \sim p$ is an absurdity
  - $(p \Rightarrow q) \wedge \sim p$ is a contingency since some cases evaluate to true and some to false.

Contingency Example

The statement $(p \Rightarrow q) \wedge (p \vee q)$ is a contingency

p	q	$p \Rightarrow q$	$p \vee q$	$(p \Rightarrow q) \wedge (p \vee q)$
T	T	T	T	T
T	F	F	T	F
F	T	T	T	T
F	F	T	F	F

Logically equivalent

- Two propositions are ***logically equivalent*** or ***simply equivalent*** if $p \Leftrightarrow q$ is a tautology.
- Denoted $p \equiv q$

Example of Logical Equivalence

Columns 5 and 8 are equivalent, and therefore, p “if and only if” q

p	q	r	$q \wedge r$	$p \vee (q \wedge r)$	$p \vee q$	$p \vee r$	$(p \vee q) \wedge (p \vee r)$	$p \vee (q \wedge r) \Leftrightarrow (p \vee q) \wedge (p \vee r)$
T	T	T	T	T	T	T	T	T
T	T	F	F	T	T	T	T	T
T	F	T	F	T	T	T	T	T
T	F	F	F	T	T	T	T	T
F	T	T	T	T	T	T	T	T
F	T	F	F	F	T	F	F	T
F	F	T	F	F	F	T	F	T
F	F	F	F	F	F	F	F	T

Additional Properties
$(p \Rightarrow q) \equiv ((\sim p) \vee q)$

p	q	$(p \Rightarrow q)$	$\sim p$	$((\sim p) \vee q)$	$(p \Rightarrow q) \Leftrightarrow ((\sim p) \vee q)$
T	T	T	F	T	T
T	F	F	F	F	T
F	T	T	T	T	T
F	F	T	T	T	T

Additional Properties
$(p \Rightarrow q) \equiv (\sim q \Rightarrow \sim p)$

p	q	$(p \Rightarrow q)$	$\sim q$	$\sim p$	$(\sim q \Rightarrow \sim p)$	$(p \Rightarrow q) \Leftrightarrow (\sim q \Rightarrow \sim p)$
T	T	T	F	F	T	T
T	F	F	T	F	F	T
F	T	T	F	T	T	T
F	F	T	T	T	T	T